THE SCOTS COLLEGE

YEAR 12 UNIT MATHEMATICS
HSC TRIAL EXAMINATION

AUGUST 2011

INSTRUCTIONS TO CANDIDATES:

- Reading time: 5 minutes
- Working time: 3 hours
- Write using blue or black pen (sketches can be in pencil)
- Board approved calculators may be used
- All 10 questions are of equal value(12 marks each).
- Answer each question in a separate booklet
- All necessary working should be shown in every question
- A table of standard integrals is provided
- Total marks: 120
- Weighting: 40%

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the Higher School Certificate examination.

Question 1(Start a New Booklet) (12 Marks)

(a) Find the reciprocal of $3\frac{4}{7}$ 1

(b) Evaluate $e^{-0.6}$ correct to three decimal places. 1

(c) Solve $4-5x \leq 3$ 2

(d) Evaluate $\int_0^2 3e^{2x}\,dx$ (Answer correct to 3 significant figures) 2

(e) After an 18% increase the price of a TV is \$1200. 1
What was the original price before the increase?

(f) Express $0.\dot{3}0\dot{3}$ as a fraction in lowest terms. 1

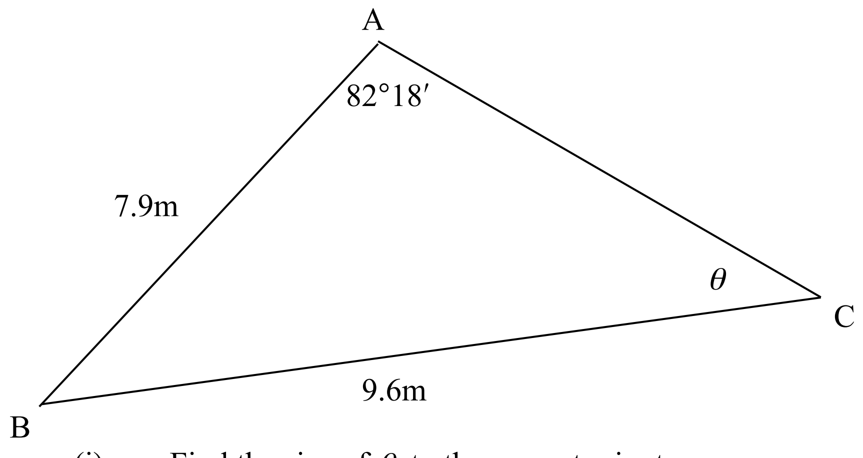
(g) Prove $\sin^2\theta\cos^2\theta+\sin^4\theta=\sin^2\theta$. 2

(h) Find a if $\sqrt{a}=\sqrt{20}+\sqrt{125}$ 2

END Q1

Question 2(Start a New Booklet) (12 Marks)

(a)



(i) Find the size of θ to the nearest minute. 2

(ii) Find the area of ΔABC to the nearest m^2. 2

(b)

If $\frac{d^2y}{dx^2}=6x-4$ and when $x=1$, $\frac{dy}{dx}=7$ and $y=12$ respectively. 2

Find y in terms of x.

(c) Evaluate $\sum_{r=2}^{6} 12 - 2r$ 2

(d) Differentiate with respect to x:

(i) $\tan 3x$ 1

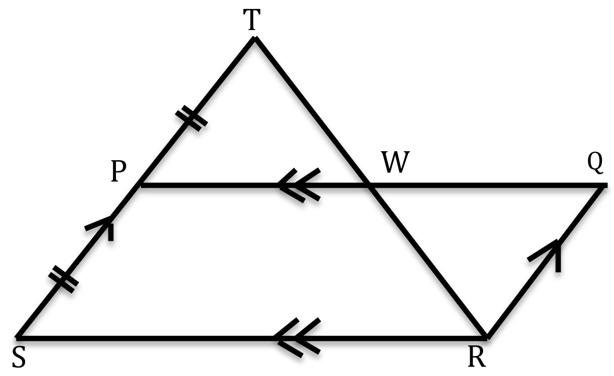
(ii) $(2x+1)^5$ 1

(iii) $\log_e(\sin x)$ 2

END Q2

Question 3(Start a New Booklet) (12 Marks)

(a)



In the above diagram, P is the midpoint of TS, PQ is parallel to SR and TS is parallel to QR.

(i) Prove $\Delta TPW \equiv \Delta WQR$ 2

(ii) Hence state why W is the midpoint of PQ. 1

(b) Find $\lim_{x \to 2} \frac{x-3}{x^2-9}$ 2

(c) Evaluate $\int_0^4 \frac{9x^2}{3+x^3} dx$ correct to 2 decimal places. 3

(d) For the equation $2x^2 + x - 3 = 0$ with roots α and β find the value of

(i) $\alpha\beta$ 1

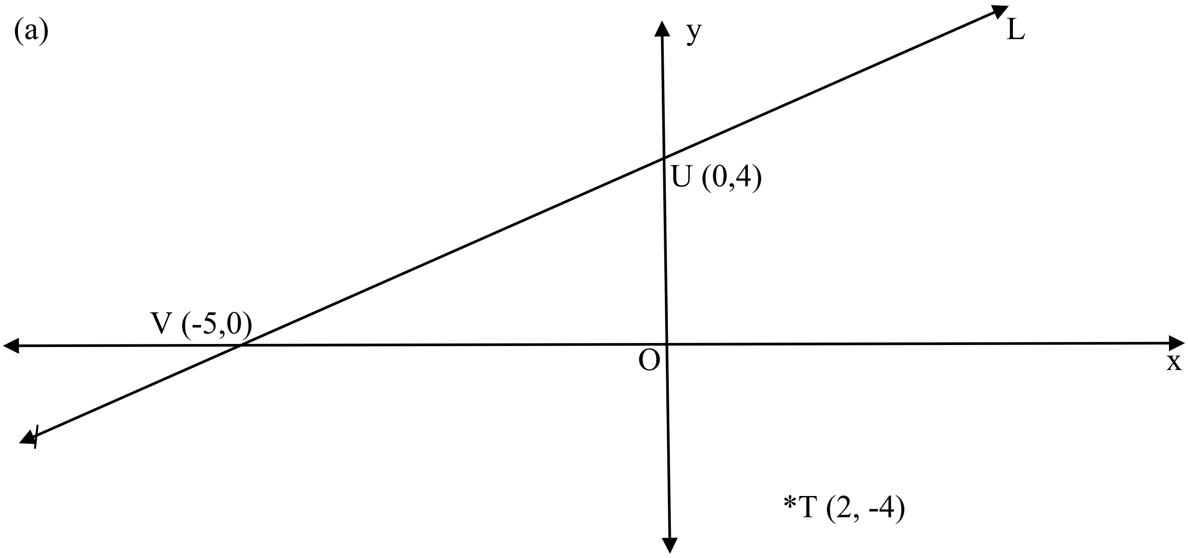
(ii) $\alpha + \beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

END Q3

Question 4(Start a New Booklet) (12 Marks)

(a)



The line L crosses the x axis at V (-5, 0) and the y axis at U (0,4). The point T (2, -4) is also shown. O is the origin.

(i) Find the gradient of the line L. 1

(ii) Show the equation of the line L is 4x - 5y + 20 = 0 2

(i) Find the perpendicular distance from the point T(2,-4) to the line L. (Leaving your answer in exact form) 2

(iv) Find the distance VU. (Leaving your answer in exact form) 1

(v) Find the area of triangle VUT. 1

(vi) At what angle is the line L inclined to the positive x axis? (Answer to the nearest minute) 2

(b) Find A, B and C if $2x^2 - 5x + 7 \equiv 2A(x+1)^2 + B(x+1) + C$ 3

END Q4

Question 5(Start a New Booklet) (12 Marks)

(a) For the parabola with equation $x^2 = 25y$ at the point (5,1) on it find:

(i) The gradient of the tangent 1

(ii) The equation of the tangent 2

(b) For the curve $y = x^3 - 3x^2 - 9x + 15$

(i) Find the y intercept. 1

(ii) Find the turning points determining their nature. 4

(iii) Find and test the point of inflexion. 2

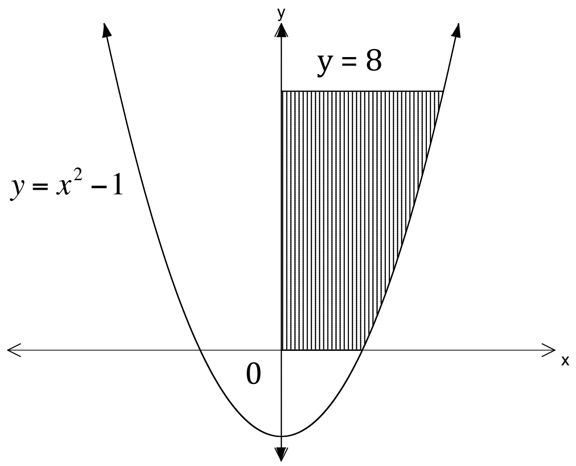
(iv) Sketch the curve clearly showing the above features. 2

END Q5

Question 6(Start a New Booklet) (12 Marks)

(a) Prove that $\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta} = 2\tan\theta$ 3

(b) The diagram shows the region bounded by the curve $y = x^2 - 1$, the line $y = 8$ and the *x* and *y* axes.



Find the volume of the solid of revolution formed when the region is rotated about the *y* axis. (Leave your answer in exact form) 3

(c) Use Simpson's rule with 3 function values to find a 2 decimal place approximation to $\int_1^7 \sqrt{x}\, dx.$ 3

(d) Find $\int (4x+1)^2 dx$ 3

END Q6

Question 7(Start a New Booklet) (12 Marks)

(a) Nic is training for a local marathon.
He has trained by completing practice runs over the marathon course.
So far he has completed three practice runs with times shown below.

Week 1	Week 2	Week 3
3 hours	2 hours 51 minutes	2 hours 42 minutes 27 seconds

(i) Show that these times form a geometric series with a common ratio $r = 0.95$. 1

(ii) If this series continues, what would be his expected time in Week 5, to the nearest second? 1

(iii) How many hours, minutes and seconds (to the nearest second) will he have run in total in his practice runs in these 5 weeks? 2

(b) For the parabola with equation $x^2 + 4 = 4y$ find the;

(i) Vertex 1

(ii) Focal Point 1

(c) Given that the limiting sum of the series 2

$1 - 2x + 4x^2 - 8x^3 + ...$ is $\frac{3}{5}$, find x.

(d) In radioactive elements, the rate of decay is proportional to the mass present given by the formulae $M = M_0e^{kt}$.

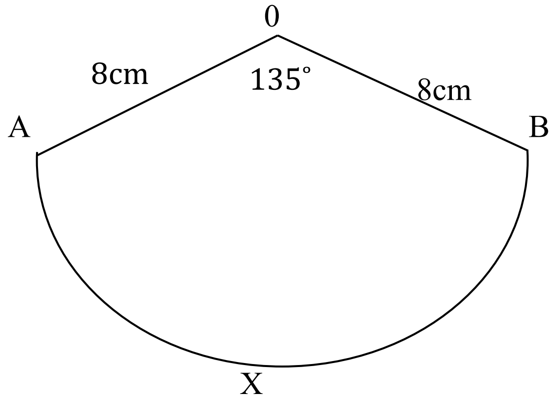
(i) If it takes 300 years for the mass of a piece of radium to decrease from 10grams to 6 grams, find the value of k to 5 decimal places. 2

(ii) Find the half-life of the material, to the nearest year. 2
(Half-life is the time taken for the element to lose half its mass)

END Q7

Question 8(Start a New Booklet) (12 Marks)

(a)

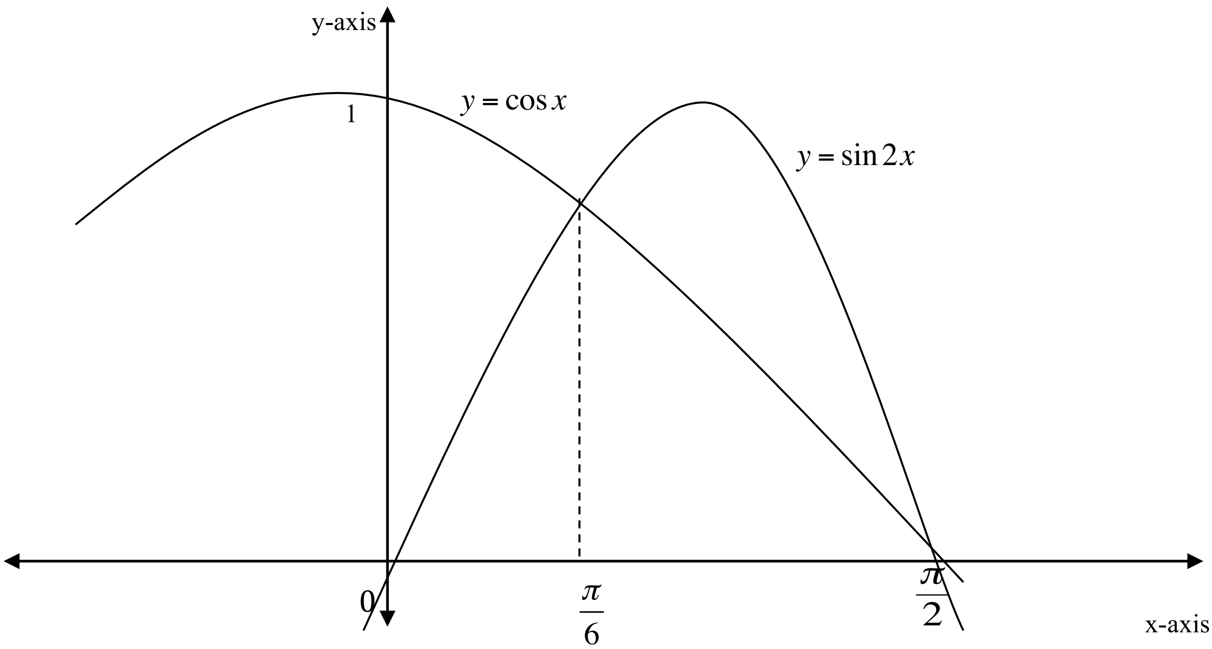


A piece of paper is in the shape of a sector of a circle. The radius is 8cm and the angle at the centre is 135º. The straight edges of the sector are placed together so that a cone is formed.

(i) Find the exact value of the arc length AXB. 1

(ii) Find the length of radius of the base of the cone. 2

(b) The diagram below shows the graphs of $y = \sin 2x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$. The two graphs intersect at $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$.



Calculate the area of the region enclosed by the x-axis and the curves y = cosx and y = sin2x. 4

(c) A dad has 5 tickets in a footy raffle in which there are two separate prizes to be won and 100 tickets are sold.

(i) Draw a tree diagram representing the outcomes of this raffle. 1

(ii) Find the probability that the dad wins only second prize. 2

(iii) Find the probability that the dad wins both prizes. 2

END Q8

Question 9(Start a New Booklet) (12 Marks)

(a) A car dealership has a car for sale for a cash price of $20 000. It can also be bought on terms over three years. The first six months are interest free and after that interest is charged at the rate of 1% per month on that months balance. Repayments are to be made in equal monthly installments beginning at the end of the first month.

A customer buys the car on these terms and agrees to monthly repayments of $\$M$. Let $\$A_n$ be the amount owing at the end of the nth month.

(i) Find an expression for A_6. 1

(ii) By first finding A_7 ,show that :

$$A_8 = (20\,000 - 6M)1{\cdot}01^2 - M(1 + 1{\cdot}01)$$ 2

(iii) Find an expression for A_{36}. 2

(ii) Assuming the car is paid off after 36 months, find the value of M. (Answer correct to the nearest $) 3

(b) A particle moves along the x-axis, its distance x cm from the origin being given by $x = 2\sin\frac{1}{3}t$, where t is in seconds.

(i) Find the velocity equation in terms of t. 1

(ii) Sketch the velocity graph of this particle for $0 \le t \le 6\pi$. 2

(iii) Using your graph or otherwise find the <u>first time</u> the particle is stationary. 1

END Q9

Question 10(Start a New Booklet) (12 Marks)

(a) A newly married couple want to build up a deposit to buy their first house. They create a savings plan in which they deposit \$500 on the first day of each month into an account which pays a fixed rate of interest of 6% per annum, compounded monthly. They start this savings plan on the first of January 2009 and hope to take the money out at the end of December 2011.

(i) How much will be in the account after 1 month? 1

(ii) Show that the amount they have in the account by the end of February 2009 is $A_2 = 500(1.005^2 + 1.005)$ 2

(iii) How much will they have in the account at the end of December 2011? 2

(b) John, Sam and William enter a golf tournament and the probabilities that each will win are $\frac{1}{5}, \frac{2}{7}$ and $\frac{3}{10}$ respectively. Assuming there is an outright winner of the tournament, find the chance (in lowest fractional form) that it is one of these three golfers. 2

(c) A rectangular sheet of metal measures 200cm by 100cm. Four equal squares with side lengths x cm are cut out of all the corners and then the sides of the sheet are turned up to form an open rectangular box.

(i) Draw a diagram representing this information. 1

(ii) Show that the volume of the box can be represented by the equation $V = 4x^3 - 600x^2 + 20\ 000x$ 1

(iii) Find the value of x such that the volume of the tool box is a maximum. (Answer to the nearest cm) 3

END Q10

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Q1

(a) $\frac{7}{25}$

(b) $e^{-0.6} = 0.549$ (3dp)

(c) $4 - 5x \le 3$

$4 - 3 \le 5x$

$x \ge \frac{1}{5}$

(d) $\int_0^2 3e^{2x}\,dx = \left[\frac{3}{2}e^{2x}\right]_0^2$

$= \frac{3}{2}(e^4 - e^0)$

$= \frac{3}{2}e^4 - \frac{3}{2}$

$= 80.4$ (3sf)

(e) $1.18 \times x = 1200$ x = original price

$x = \frac{1200}{1.18}$

$x = \$1016.95$

(f) $x = 0.\dot{3}0\dot{3}$

$1000x = 303.\dot{3}0\dot{3}$

$999x = 303$

$x = \frac{303}{999}$

$x = \frac{101}{333}$

(g) Prove $\sin^2\theta\cos^2\theta + \sin^4\theta = \sin^2\theta$

$\sin^2\theta(\cos^2\theta + \sin^2\theta) = \sin^2\theta$

$\sin^2\theta(1) = \sin^2\theta$

$\therefore \sin^2\theta = \sin^2\theta$ as req.

(h) $\sqrt{a} = \sqrt{20} + \sqrt{125}$

$\sqrt{a} = 2\sqrt{5} + 5\sqrt{5}$

$\sqrt{a} = 7\sqrt{5}$

$\sqrt{a} = \sqrt{49 \times 5}$

$\sqrt{a} = \sqrt{245}$

$\therefore a = 245$

Q2 (a)

(i) $\frac{\sin\theta}{7.9} = \frac{\sin 82^\circ 18'}{9.6}$

$\sin\theta = \frac{7.9 \times \sin 82^\circ 18'}{9.6}$

$\theta = 54^\circ 38'$

(ii) Area $\Delta ABC = \frac{1}{2} \times 7.9 \times 9.6 (\sin 43^\circ 4'$

$\doteqdot 26\ m^2$

(b) $\frac{d^2y}{dx^2} = 6x - 4$

$\frac{dy}{dx} = 3x^2 - 4x + C$

$7 = 3 - 4 + C$

$7 = -1 + C$

$C = 8$ $\frac{dy}{dx} = 3x^2 - 4x + 8$

$y = x^3 - 2x^2 + 8x + C$

$12 = 1 - 2 + 8 + C$

$12 = 7 + C$

$C = 5$

$\therefore y = x^3 - 2x^2 + 8x + 5$

(c) $\sum_{r=2}^{6} 12 - 2r = 8 + 6 + 4 + 2 + 0$

$= 20$

(d) (i) $\frac{d}{dx}\tan 3x = 3\sec^2 3x$

(ii) $\frac{d}{dx}(2x+1)^5 = 5 \times 2(2x+1)^4$

$= 10(2x+1)^4$

(iii) $\frac{d}{dx}\log_e(\sin x) = \frac{\cos x}{\sin x}$

$= \cot x$

1, 2, 3 } DS 6, 7, 8 } PA

4, 5 } PR 9, 10 } TG.

Q3 (a) (i) $\angle TWP = \angle QWR$ (vert opp)
$\angle PTR = \angle QRW$ (alt $\angle$ between ll-lines) ✓
PQRS is a parallelogram. PQ‖SR, PS‖QR
PS = QR (opp sides of parallelogram) ✓
P is given Midpoint, ∴ PS = ~~QR~~ = PT, hence by AAS $\triangle TPW \equiv \triangle WQR$ ✓
(ii) By corresponding sides of congruent triangles PW = WQ ✓
hence W is midpoint of PQ ✓

(b) $\lim_{x \to 2} \frac{x-3}{x^2-9} = \lim_{x \to 2} \frac{(x-3)}{(x-3)(x+3)}$ ✓
$= \frac{1}{5}$ ✓

(c) $\int_0^4 \frac{9x^2}{3+x^3} = \left[3\ln(3+x^3)\right]_0^4$ ✓
$= 3(\ln 67 - \ln 3)$
$= 9.32$ ✓

(d) $2x^2 + x - 3 = 0$
(i) $\alpha\beta = -\frac{3}{2}$ ✓
(ii) $\alpha + \beta = -\frac{1}{2}$ ✓
(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓
$= \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{3}{2}\right)$
$= \frac{1}{4} + 3$
$= 3\frac{1}{4}$ ✓

Q4 (a) (i) $m_L = \frac{4-0}{0--5} = \frac{4}{5}$ ✓
(ii) $y - 4 = \frac{4}{5}(x-0)$ ✓
$5y - 20 = 4(x-0)$
$5y - 4x - 20 = 0$
$4x - 5y + 20 = 0$ ✓ as req.

(iii) perp dist $= \frac{|4(2) - 5(-4) + 20|}{\sqrt{4^2 + (-5)^2}}$ ✓
$= \frac{|8 + 20 + 20|}{\sqrt{16+25}}$
$= \frac{48}{\sqrt{41}}$ ✓

(iv) dist VU $= \sqrt{(-5-0)^2 + (0-4)^2}$
$= \sqrt{25 + 16}$
$= \sqrt{41}$ ✓

(v) Area $\triangle VUT = \frac{1}{2} \times \sqrt{41} \times \frac{48}{\sqrt{41}}$
$= 24\ u^2$ ✓

(vi) $\tan\theta = \frac{4}{5}$ ✓
$\theta = \tan^{-1}\left(\frac{4}{5}\right)$
$\therefore \theta = 38°40'$ ✓

(b)
$2x^2 - 5x + 7 \equiv 2A(x+1)^2 + B(x+1) + C$
$A = 1$.
$2x^2 - 5x + 7 = 2(x+1)^2 + B(x+1) + C$
Let $x = -1$.
$2 + 5 + 7 = 0 + 0 + C$
$\therefore C = 14$
$2x^2 - 5x + 7 = 2(x+1)^2 + B(x+1) +$
Let $x = 0$
$7 = 2 + B + 14$
$7 - 16 = B$
$B = -9$

Hence $A = 1$, $B = -9$, $C = 14$. ✓ ✓ ✓

Q5 (a) $x^2 = 25y$

$y = \frac{x^2}{25}$

(i) $\frac{dy}{dx} = \frac{2}{25}x$.

$m_t = \frac{2}{25}(5)$

$m_t = \frac{10}{25}$

$m_t = \frac{2}{5}$

(ii) $y - 1 = \frac{2}{5}(x - 5)$

$y - 1 = \frac{2}{5}x - 2$

$y = \frac{2}{5}x - 1$

(or)

$5y = 2x - 5$

$2x - 5y - 5 = 0$

(b) $y = x^3 - 3x^2 - 9x + 15$

(i) y-intercept is 15

(ii) $y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

let $y' = 0$ for stat pts.

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3,\ x = -1.$

when $x = 3$, $y = 27 - 27 - 27 + 15$

$y = -12$

when $x = -1$, $y = -1 - 3 + 9 + 15$

$y = 20$

∴ Stat pts $(3, -12)$ $(-1, 20)$.

Testing when $x = 3$

$y'' = 18 - 6$

$y'' = 12$ concave up

∴ Min TP at $(3, -12)$

Testing when $x = -1$

$y'' = -6 - 6$

$y'' = -12$

concave down

∴ Max TP at $(-1, 20)$

(iii) let $6x - 6 = 0$ (for pt of inflection)

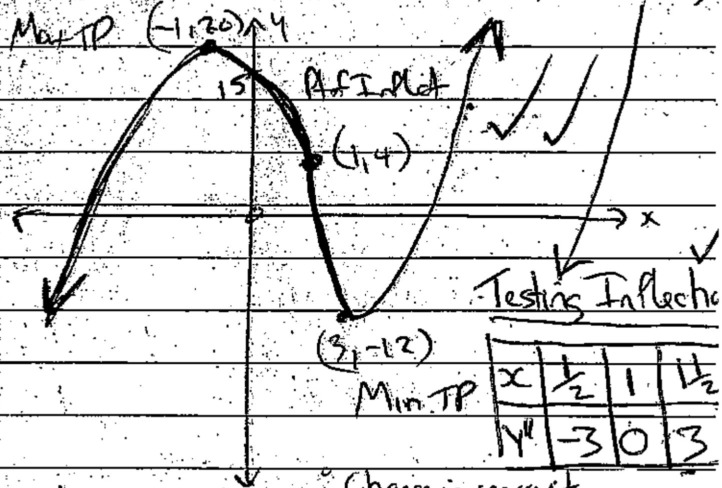
$6x = 6$

$x = 1.$

$y = 1 - 3 - 9 + 15$

$y = 4.$

∴ Pt of inflection at $(1, 4)$



Testing Inflection

x	$\frac{1}{2}$	1	$1\frac{1}{2}$
y''	-3	0	3

∴ Change in concavity

Q6 (a) Prove $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2\tan\theta$

$\frac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{1-\sin^2\theta} = 2\tan\theta$

$\frac{\cos\theta + \cos\theta\sin\theta - \cos\theta + \cos\theta\sin\theta}{\cos^2\theta} = 2\tan\theta$

$\frac{2\cos\theta\sin\theta}{\cos^2\theta} = 2\tan\theta$

$\frac{2\sin\theta}{\cos\theta} = 2\tan\theta$

∴ $2\tan\theta = 2\tan\theta$ as req.

P3/6

(b) $y = x^2 - 1$

$y + 1 = x^2$ ✓

$V = \pi \int_0^8 y + 1 \, dy$ ✓

$= \pi \left[\frac{y^2}{2} + y \right]_0^8$ ✓

$= \pi [(32 + 8) - 0]$ ✓

$= 40\pi \ u^3$

(c)

x	1	4	7
y	1	2	$\sqrt{7}$
	y_0	y_1	y_2

$h = \frac{7-1}{2}$ ✓

$h = 3$ ✓

$\int_1^7 \sqrt{x} \, dx \doteq \frac{3}{3} \left(1 + \sqrt{7} + 4(2) \right)$ ✓

$\doteq 1 (1 + \sqrt{7} + 8)$

$\doteq 9 + \sqrt{7}$

$\doteq 11.65$ ✓

(d) $\int (4x+1)^2 \, dx = \frac{(4x+1)^3}{12} + c$ ✓ ✓

must have c to gain 2 marks

Q7 (a) (i) $\frac{2.85}{3} = 0.95$ ✓

$\frac{2.7075}{2.85} = 0.95$ OR similar method.

(ii) $T_5 = 2.7075 \times 0.95 \times 0.95$

$= 2\text{hr } 26\text{min } 37\text{sec}$ ✓

(iii) $S_5 = \frac{3(1 - 0.95^5)}{1 - 0.95}$ ✓

$= 13\text{hr } 34\text{min } 23\text{sec}$ ✓

(b) $x^2 + 4 = 4y$

$x^2 = 4y - 4$

$x^2 = 4(y-1)$

$x^2 = 4(1/4)(y-1)$

(i) Vertex $(0, 1)$ ✓

(ii) Focal pt $(0, 1\frac{1}{4})$ ✓

(c) $\frac{a}{1-r} = \frac{3}{5}$

$a = 1$

$r = -2x$ ✓

$\therefore \frac{1}{1 - -2x} = \frac{3}{5}$

$\frac{1}{1+2x} = \frac{3}{5}$

$5 = 3 + 6x$

$2 = 6x$ ✓

$x = \frac{1}{3}$

(d) $M = M_0 e^{kt}$

(i) $6 = 10 e^{300k}$ ✓

$0.6 = e^{300k}$

$300k = \ln 0.6$

$k = \frac{\ln 0.6}{300} = -0.00170$

(ii) $1 = 2e^{-0.00170t}$

$0.5 = e^{-0.00170t}$ ✓

$-0.00170t = \ln 0.5$

$t = \frac{\ln 0.5}{-0.00170}$

$t = 407.7$ ✓

$t = 408$ years

P4/6

8. (a) Q 8

(a)

(i)

Arc Length $AXB = 8 \times \frac{3\pi}{4}$

$= 6\pi$ cm ✓

(ii)

$C = 2\pi r$

$6\pi = 2\pi r$ ✓

$\frac{6\pi}{2\pi} = r$

$r = 3$ cm ✓

(b)

$A = \int_0^{\frac{\pi}{6}} \sin 2x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \, dx$ ✓ ✓

$= \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}} + \left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ ✓

$= -\frac{1}{2}\left[\cos\frac{\pi}{3} - \cos 0\right] + \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right]$

$= -\frac{1}{2}\left[\frac{1}{2} - 1\right] + \left[1 - \frac{1}{2}\right]$

$= \frac{1}{4} + \frac{1}{2}$

$= \frac{3}{4} u^2$ ✓

(c)

(ii) P(Only wins second) $= \frac{95}{100} \times \frac{5}{99}$ ✓

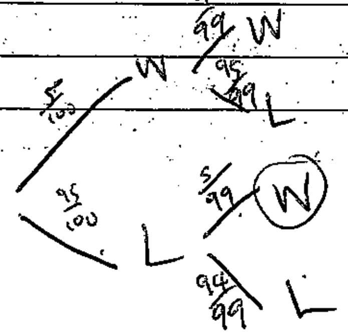
$= \frac{19}{396}$ ✓

$= 0.0479$

(iii) P(Wins Both Prizes) $= \frac{5}{100} \times \frac{4}{99}$ ✓

$= \frac{1}{495}$ ✓

(i)



✓

Q9 (a) (i) $A_6 = 20\,000 - 6M$ ✓

(ii) $A_7 = (20\,000 - 6M)1.01 - M$

$A_8 = [(20\,000 - 6M)1.01 - M]1.01 - M$ ✓

$= (20\,000 - 6M)(1.01)^2 - 1.01M - M$

$\therefore A_8 = (20\,000 - 6M)1.01^2 - M(1 + 1.01)$ ✓

(iii) ✓ ✓

$A_{36} = (20\,000 - 6M)1.01^{30} - M(1 + 1.01 + \ldots + 1.01^{29})$

(iv) $A_{36} = 0$ ✓

$0 = (20\,000 - 6M)1.01^{30} - M\left(\frac{1 \cdot (1.01)^{30} - 1}{1.01 - 1}\right)$

$\frac{M(1.01^{30} - 1)}{0.01} = 20\,000(1.01)^{30} - 6M(1.01)^{30}$

$M\left(\frac{1.01^{30} - 1}{0.01}\right) + 6M(1.01)^{30} = 20\,000(1.01)^{30}$

$M = \frac{20\,000(1.01)^{30}}{\frac{1.01^{30} - 1}{0.01} + 6(1.01)^{30}}$ ✓

$\therefore M = \$628.77$

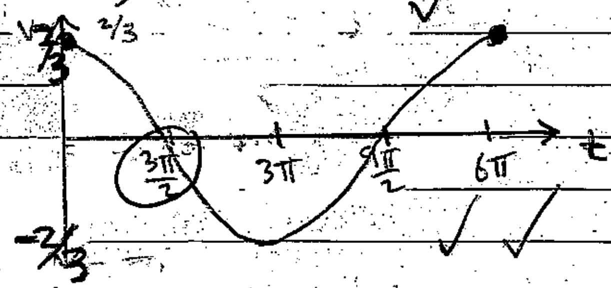
$M = \$629$ (nearest \$) ✓

(b) $x = 2\sin\frac{1}{3}t$

$v = x' = \frac{2}{3}\cos\frac{1}{3}t$

(i) $V = \frac{2}{3}\cos\frac{1}{3}t$ ✓

(ii)



✓ ✓

(iii) Particle is 1st stationary $V = 0$, when $t = \frac{3\pi}{2}$ sec. ✓ P5/6

Q10

$r = \frac{0.06}{12} = 0.005$

(a) (i) $A_1 = 500(1.005)$

$= \$502.50$ ✓

(ii) $A_2 = 500(1.005)(1.005) + 500(1.005)$ ✓

$= 500(1.005)^2 + 500(1.005)$

$= 500(1.005^2 + 1.005)$ ✓

as req ✓

(iii) $A_{36} = 500(1.005^{36} + 1.005^{35} + \dots + 1.005^1)$ ✓

$= \frac{500(1.005)(1.005^{36} - 1)}{0.005}$ ✓

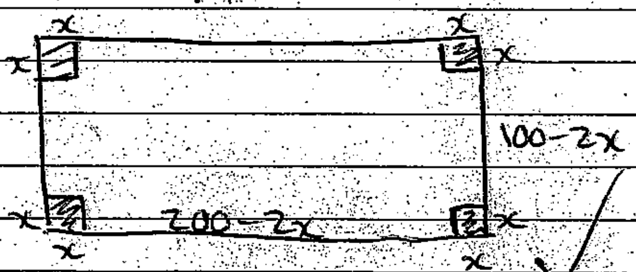
$A_{36} = \$19\ 766.39$ ✓

(b) P(Either John, Sam or William Wins)

$= \frac{1}{5} + \frac{2}{7} + \frac{3}{10}$

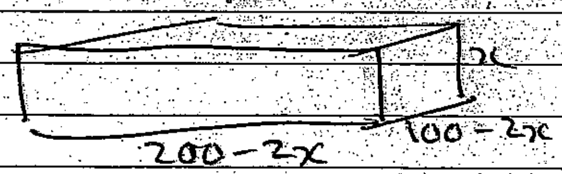
$= \frac{11}{14}$

(c) (i)



✓

OR



(ii) $V = x(100-2x)(200-2x)$

$\therefore V = 4x^3 - 600x^2 + 20000x$ ✓

(iii)

$V' = 12x^2 - 1200x + 20\ 000$

let $V' = 0$ for stat pts

$6x^2 - 600x + 10\ 000 = 0$

$x = \frac{600 \pm \sqrt{600^2 - 4 \times 6 \times 10000}}{12}$ ✓ ✓

$x = 78.87$, $x = 21.13$

$V'' = 24x - 1200$

Testing

When $x = 78.87$

$V'' = 692$ ∴ Concave up ∪

Min TP at $x = 78.8$

When $x = 21.13$

$V'' = -692$ ∴ Concave down ∩

Max TP at $x = 21.13$

∴ Max Vol achieved when $x = 21$ cm ✓

p 6/6